Methods Of Using Historical and Scientific Materials in Mathematics Education

Bahadur Omar Oghly Tahirov
Baku State University

Shahin Mutarif Oghly Aghazade
Baku State University

Abstract

Modern society increasingly demands graduates equipped with a deep worldview and a solid foundation of knowledge, skills, and competencies. Throughout their education, students are exposed to the fundamentals of various sciences, including mathematics, for eleven years. Given that mathematics is one of the core subjects in secondary education, it is imperative that students not only master the methods of computation and logical reasoning but also deeply engage with mathematical concepts through historical contexts. This paper explores the integration of historical and scientific materials into mathematics education, emphasizing their significance in enriching the learning experience. By utilizing historical facts and narratives, educators can present mathematics as a dynamic and evolving discipline, fostering a sense of connection between students and the subject matter. The research indicates that the incorporation of historical materials enhances students' conceptual understanding, critical thinking skills, and mathematical modeling abilities. Furthermore, it promotes a broader appreciation for mathematics as an essential component of personal culture and societal development. This study aims to provide educators and prospective students with strategies to leverage historical resources, thereby transforming mathematics into a vibrant field of inquiry. Through this approach, we seek to cultivate a generation of learners who view mathematics not only as a set of abstract concepts but also as a rich, contextualized discipline that reflects human thought and innovation over time.

Keywords: History of mathematics, content of historical and scientific materials, geometric methods of solving equations, proof of identities using geometric methods, lune and its area, content requirements, geometric methods of solving equations, proof of identities using geometric methods.

AMS Subject Classification: 97E50 - Mathematics education, 97C30 - Curriculum and pedagogy in mathematics, 97D40 - Teaching methods and instructional strategies, 01A30 - History of mathematics, 97K10 - Use of technology in teaching mathematics.

Introduction

The use of certain facts from the history of the development of mathematics in the process of teaching mathematics allows students to see a broad picture of the emergence and development of mathematics, thereby creating a bridge between mathematics and universal human culture. The use of materials from the history of mathematics in mathematics education contributes to turning mathematical knowledge into an important component of each person's personal culture [1] [2]. Integrating historical and practical contexts into the school mathematics curriculum is realized by forming students' understanding of

[1]Малыгин, К. А., "Элементы историзма в преподавании математики в средней школе: Пособие для учителей", Просвещение, 2013.

[2] Треьякова, О. А., "Использование заданий историко-математического содержания в старшей школе", (51)(393) Молодой ученый, 2022, p. 407-408.

mathematics as part of universal human culture[3] [4]. Therefore, the organic supplementation of the content of the school mathematics course with specific materials from the history of mathematics serves to familiarize students with the struggle of ideas, the destinies of great discoveries, and the names of the people who developed mathematics.

The problem of using historical and scientific materials in mathematics education has always been relevant and remains so.

In existing textbooks and teaching aids, little space is devoted to historical and scientific material, as a result of which students view mathematics as "abstract reality" rather than as the result of the labor of many generations of researchers[5].

This is truly an interesting point of view. If you look at the history of mathematics, you can see how it developed as a result of the practical needs and challenges faced by humanity. For example, the foundations of algebra were developed to solve problems in trade and agriculture; geometry was necessary for measuring land and construction, while arithmetic was essential for accounting and trade operations.

1. Literature Review

Thus, mathematics was born from practical use and applied to solve everyday problems. This cycle of using, analyzing, systematizing, and applying knowledge again helped humanity improve its methods and expand its capabilities[6].

Therefore, mathematics continues to develop for two main reasons:

1. Due to Practical Necessity:

Mathematics has historically emerged from the practical need to solve problems faced by humanity. These necessities have made the application of mathematical methods essential in everyday life, commerce, engineering, science, and many other fields. For example, ancient civilizations developed mathematical concepts in response to the calculation requirements of agriculture, trade, and construction.

In today's world, these practical necessities have become more complex. In modern society, mathematical competencies are required in various areas, including data analysis, financial modeling, and engineering design, as well as in addressing environmental issues. The advancements in big data and data science, in particular, necessitate the use of mathematical models and algorithms to make sense of large amounts of data and optimize decision-making processes. Additionally, everyday challenges often lead to mathematical inquiries in education, healthcare, and social sciences, ensuring that mathematics remains in a state of continuous development. For instance, the analysis of statistical data in healthcare allows for understanding disease spread trends and optimizing treatment methods through mathematical approaches.

2. Due to the Inherent Needs Within Mathematics Itself:

[3] Круликовский, Н. Н., "Сообщение сведений из истории математики в средней школе", В Л. Ф. Пичурин (Сост.), Воспитание учащихся при обучении математике, Просвещение, 1987, p. 28-31.

[4] Алешкова, Т. Н., "Гуманитаризация общего математического образования", 1(5) Математические структуры и моделирование, 2000, p. 155-161.

[5] Рыбников, К. А., "Об историко-методологических основах математического образования учителей", 3 Математика в школе, 1982, p. 48-49.

[6] Алексеева, В. А., "Методика отбора и использования историко-научного материала в процессе обучения математике в школе" (Диссертация), 1998.

Mathematics evolves not only because of external practical requirements but also due to its internal dynamics and needs. The in-depth exploration of mathematical theories leads to the emergence of new concepts, methods, and theories. Mathematical research is characterized by a constant effort to push the boundaries of existing theories and open new areas of study. For example, a specific mathematical problem might lead to the development of solutions in another domain, resulting in the birth of new theories and concepts in the process.

These inherent needs are also fueled by an increasing interest in mathematical thinking itself. Mathematicians tackle previously unresolved or partially understood problems, developing new methods and approaches in the process. For instance, the study of complex numbers and abstract mathematical concepts like fractal geometry has deepened mathematical thought and found applications in other scientific disciplines. Furthermore, the inherently abstract nature of mathematical thinking encourages constant self-reflection and evolution within the field. This not only allows for the deepening and enriching of mathematical theories but also leads to the discovery of new application areas.

Finally, it should be noted that the inclusion of historical and scientific materials in mathematics education allows students to form a complete picture of the development of mathematics and enhance their mathematical thinking[7].

For example, lessons can include examples of real mathematical problems solved by scientists in the past, as well as examples of modern applications of mathematics in various fields, such as machine learning, cryptography, finance, etc. These examples will help students see how mathematics is used to solve pressing problems and how it continues to evolve along with our world[8].

In 1950, the academician A. I. Markushevich pointed out the lack of historical materials in school textbooks. Since then, attempts have been made to integrate historical and scientific materials into the content of school mathematics courses. However, even in modern textbooks, this aspect does not always receive sufficient attention[9].

In 1975, a significant initiative was introduced in the journal *Mathematics in School* with the section titled "Mathematical Calendar." This section was designed to bridge the gap between mathematical theory and its historical context, aiming to enrich the curriculum with engaging content that reflects the evolution of mathematical thought.

The "Mathematical Calendar" provided a platform for educators to explore the historical narratives surrounding key mathematical concepts and figures. By showcasing important events, discoveries, and milestones in the history of mathematics, this initiative emphasized that mathematics is not just a collection of abstract theories but a dynamic discipline shaped by human creativity and cultural development. This historical perspective helps to contextualize mathematical principles, allowing students and educators to appreciate the subject's relevance and significance in various real-world applications.

Furthermore, by incorporating historical anecdotes and the stories of mathematicians, the initiative facilitated a more relatable and accessible approach to learning mathematics. Students often find abstract concepts challenging; however, by connecting these concepts to their historical origins and applications, educators can inspire a sense of curiosity and engagement among learners. This

[7] Jankvist, U. T., A categorization of the “whys” and “hows” of using history in mathematics education. Educational Studies in Mathematics, 71(3), 2009, 235-261.

[8] Степура, Д. А., "Применение исторического материала на уроках математики и во внеурочное время", 1 Дневник науки, 2021, p. 1-9.

[9] Маркушевич, А. И., "О повышении идейно-теоретического уровня преподавания математики в средней школе", 1 Математика в школе, 1950, p. 1-4.

contextualization also encourages critical thinking, as students begin to see how mathematical ideas have evolved over time in response to societal needs and intellectual challenges.

Overall, the "Mathematical Calendar" initiative played a crucial role in transforming the teaching and learning of mathematics, fostering a deeper understanding of the subject, and making it more appealing to students. It encouraged educators to adopt a more holistic approach to mathematics education, one that recognizes the interplay between history and mathematical theory, ultimately enriching the learning experience for students.[10]

Mathematics education has been shaped by various approaches and methodologies throughout history. In this context, the use of historical and scientific materials in mathematics education enriches learning processes and provides students with a deeper understanding. For instance, Bishop suggests that utilizing historical materials in mathematics learning demonstrates the evolutionary process of mathematics to students, thereby enhancing their conceptual understanding. Similarly, Kaiser and Blömeke emphasize that addressing historical topics in mathematics teaching improves students' mathematical thinking skills[11] [12].

The role of historical materials in education not only facilitates the understanding of concepts but also aids students in grasping the societal and cultural dimensions of mathematics. Sullivan states that teaching mathematical concepts in a historical context helps students better comprehend these concepts. Additionally, Baker reveals that the historical dimensions of mathematics education play a significant role in developing students' mathematical modeling and problem-solving skills[13] [14].

The literature presents various suggestions and models for integrating historical materials into mathematics education. Fraser indicates that systematically incorporating historical materials into curricula can enhance student motivation. Moreover, Lambdin and Smith highlight the importance of teachers having adequate knowledge and skills in this area for the effective use of historical materials[15] [16]

Indeed, students often perceive mathematics as a static and finished set of knowledge, whereas it is actually an ever-evolving field of science. To help students better understand this aspect, more interactive teaching methods can be integrated into the learning process, such as using practical problems, research projects, and historical cases.

In conclusion, the literature demonstrates that the integration of historical and scientific materials in mathematics education positively impacts student learning processes. In this regard, the aim of our study is to deeply investigate the role of historical and scientific materials in mathematics education, suitable to the needs of modern society, and to develop recommendations for how these materials can be used more effectively in education.

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[10] Cajori, F., A History of Mathematics. New York: Dover Publications, 1990.

[11] Bishop, A. J.., "Mathematical knowledge as a cultural phenomenon: A study of the impact of cultural factors on the teaching and learning of mathematics", 19(3) Educational Studies in Mathematics, 1988, p. 237-254.

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[13] Sullivan, P., "Teaching mathematics through history: A powerful approach", 56(4) Australian Mathematics Teacher, 2000, p. 10-14.

[14] Baker, D., "Historical perspectives in mathematics education: Implications for teaching and learning", 94(3) Mathematics Teacher, 2001, p. 160-165.

[15] Fraser, D., "Teaching mathematics through history: The role of historical sources in mathematics education", 21(3) International Studies in the Philosophy of Science, 2007, p. 317-330.

[16] Lambdin, D., V., & Smith, Norman, "Teachers' knowledge of history and its influence on the teaching of mathematics", 102(5) Mathematics Teacher, 2008, p. 344-349.

2. Methodology

Incorporating historical and scientific materials into mathematics education not only helps students gain a deeper understanding of the subject but also enhances their logical thinking and problem-solving skills. Such approaches foster students' ability to apply mathematical reasoning in real-world contexts, as highlighted in studies on improving students' critical thinking in mathematics[17]. Among the notable contributions from the 1980s and 1990s, the works of V.P. Gleizer stand out prominently[18]. His three books, aimed specifically at schoolchildren, are particularly noteworthy for their comprehensive exploration of the history of mathematics. Gleizer's objective was not merely to present historical facts; rather, he sought to equip teachers with the tools to integrate these historical insights into their lessons and extracurricular activities effectively. In this way, Gleizer aimed to create a learning environment where historical insights were not just facts but tools for deeper mathematical understanding. The incorporation of historical context into mathematics education can ignite students' curiosity and encourage them to see mathematics as a dynamic field shaped by human thought and creativity.

In parallel, the influential works of the renowned American mathematician Morris Kline, specifically Mathematics: The Loss of Certainty (Moscow, 1984) and Mathematics: The Search for Truth (Moscow, 1988), deserve special attention. Kline's writings offer a sweeping overview of the development of mathematics, tracing its roots from ancient civilizations through various pivotal moments in history to contemporary times. His exploration goes beyond mere chronology; it examines the philosophical implications of mathematical discoveries and the quest for certainty and truth in mathematical thought. By presenting this broad historical panorama, Kline encourages readers to reflect on the nature of mathematical inquiry and its relevance to both historical and modern contexts[19]

Recent psychological research underscores the benefits of incorporating historical and scientific materials in mathematics education, particularly during adolescence. This developmental stage is characterized by a growing capacity for abstract thinking and a heightened interest in understanding the world. By introducing students to the historical foundations of mathematical concepts, educators can create a more engaging and meaningful learning experience. The interplay between history and mathematics not only aids in the retention of mathematical principles but also fosters critical thinking and analytical skills.

In contemporary education, it is increasingly recognized that the teaching of mathematics should not solely focus on technical skills and problem-solving. Instead, integrating materials related to the history of mathematics can enhance the educational experience by illustrating the subject's relevance and applicability in various contexts[20]. Historical examples can serve as powerful motivators for students, providing them with a narrative that contextualizes their learning and demonstrates the human aspect of mathematical discovery.

In conclusion, the incorporation of historical materials into mathematics education is not just a pedagogical enhancement; it is a necessary evolution of the curriculum that respects the rich tapestry of mathematical development. As educators strive to cultivate a comprehensive understanding of mathematics, it becomes imperative to recognize and utilize the historical narratives that have shaped the field. By doing so, we not only honor the contributions of past mathematicians but also inspire future generations to engage with mathematics as a living, evolving discipline.

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[17] Жохов, В. А., "История математики как средство развития интереса учащихся к предмету", Математика в школе, 1985, p. 40-45.

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[20] Fauvel, J., & van Maanen, J., (Eds.) History in mathematics education: The ICMI study. Dordrecht: Kluwer Academic Publishers. 2000.

Research methods. We conducted a theoretical analysis of the well-known studies of the authors of works[21].

3. Examples of Historical and Scientific Materials[22]

1. In Al-Khwarizmi's book «Kitab al-Jabr wa al-Muqabala», a geometric solution to the equation is provided $x^2+ax=b$.

To solve the equation $x^2+ax=b$, a square is drawn, with its side length equal to $\left(x+\frac{a}{2}\right)$ and from any vertex of this square $\frac{a}{4}$, x and $\frac{a}{4}$, segments are marked off on the adjacent sides. Then, through the division points, lines parallel to the opposite sides are drawn. As a result, the square is divided into nine parts:

4 squares with side length $\frac{a}{4}$; 1 square with side length x; 4 rectangles with side lengths and $\frac{a}{4}$ и x. If the area of the square is denoted by, we have:

Thus, we obtain:

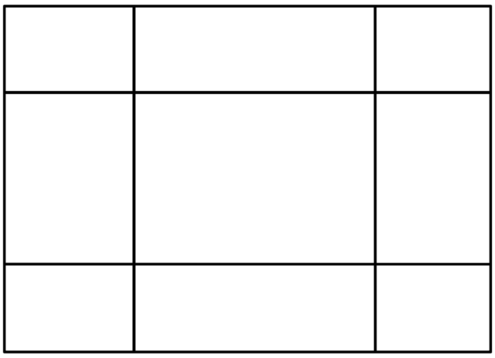

Figure 1

$$S=x^2+4\cdot\left(\frac{a}{4}\right)^2+4\cdot\frac{a}{4}\cdot x=(x^2+ax)+4\cdot\left(\frac{a}{4}\right)^2=b+\frac{a^2}{4}$$

Hence, we have: $\left(x+\frac{a}{2}\right)^2=b^2+\frac{a^2}{4}\Leftrightarrow x=-\frac{a}{2}+\sqrt{b^2+\frac{a^2}{4}}$.

$$\left(x+\frac{a}{2}\right)^2=b^2+\frac{a^2}{4}\Leftrightarrow x=-\frac{a}{2}+\sqrt{b^2+\frac{a^2}{4}}$$

[21] Katz, V. J., & Tzanakis, C., (Eds.) Recent developments on introducing a historical dimension in mathematics education. Washington, DC: Mathematical Association of America. 2011.

[22] Рыбников, К. А., История математики, МГУ, 1974.

2. To prove the identity $(a+b)^2 = a^2 + b^2 + 2ab$, draw a square with side length. From any vertex of this square, mark segments of lengths $(a+b)$ and on the adjacent sides. Then, through the division points a и b, draw lines parallel to the opposite sides. As a result, the area of the square is divided into four parts:

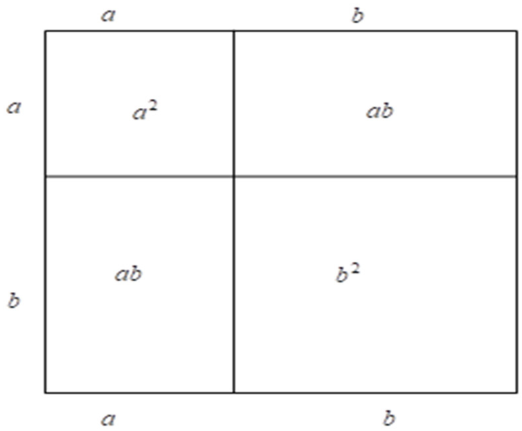

Figure 2.

1 square with side length a ;

2 rectangles with sides and a и b;

1 square with side length b.

If the area of the square is denoted by S, then:

$$S = a^2 + 2ab + b^2 \text{ or}$$

$$(a+b)^2 = a^2 + 2ab + b^2.$$

3. The first squarable figure bounded by a curved line was discovered in Ancient Greece. It was the lune. The first squarable lune was obtained from the intersection of semicircles. Following the author of the work, we will develop an interactive educational application that will help students better understand geometric concepts. In this application, students will be able to visualize the intersection of semicircles constructed on the sides of a right triangle and explore the properties of the resulting figures. The application may offer interactive tasks and games to help students consolidate their knowledge of geometry and develop logical thinking[23].

[23] Deliyianni, E., & Gagatsis, A., The development of students' geometrical thinking through interactive applications. Educational Studies in Mathematics, 86(1), 2014. 123-138.

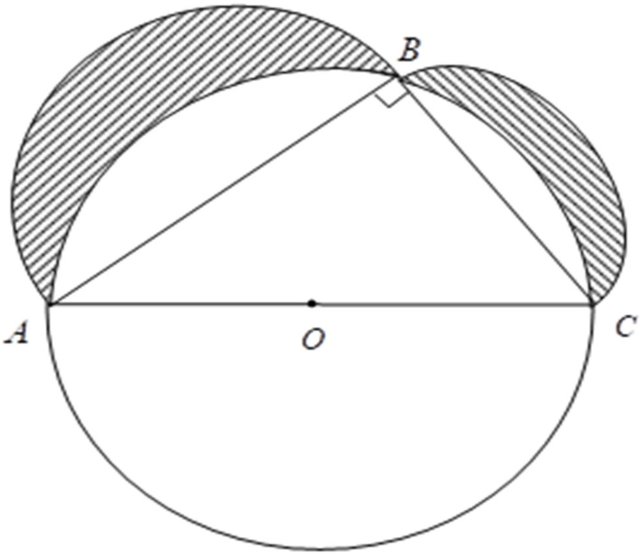

Figure 3.

Figure 3-Geometric representation of intersecting semicircles and the concept of lune area. This figure illustrates the intersection of semicircles constructed on the sides of a right triangle, demonstrating the relationship between the areas of lunes and the triangle. Annotations highlight specific properties to assist with subsequent formula explanations. It can be shown that the areas of the shaded lunes are equal to the area of triangle ABC. If we denote the area of the lune based on arc AB as S_1 S_1, the area of the lune based on arc BC as S_2 S_2, the area of the segment based on chord AB AB as x x, and the area of the segment based on chord BC BC as y y, then we obtain:

$$\frac{\pi a^2}{8} = S_1 + x; \quad \frac{\pi b^2}{8} = S_2 + y; \quad \frac{\pi c^2}{2} = \frac{ab}{2} + x + y;$$

$$x = \frac{\pi a^2}{8} - S_1 \; x = \frac{\pi a^2}{8} - S_1; \quad y = \frac{\pi b^2}{8} - S_2.$$

Therefore, $$S_1 + S_2 = \frac{ab}{2}.$$

4. In Arabic mathematics, there was significant scientific interest in operations with algebraic irrationalities. For example, eighth graders are familiar with the formulas:

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}.$$

Their validity is easy to check: Let:

$$\sqrt{a + \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} + \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} \quad \sqrt{a + \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} + \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}.$$

Then, by squaring both sides of the last equation, we obtain:

$$\left(\sqrt{a+\sqrt{b}}\right)^2=\left(\sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}\right)^2=$$

$$=\frac{a+\sqrt{a^2-b}+a-\sqrt{a^2-b}}{2}=a+\sqrt{b}$$

or $$a+\sqrt{b}=a+\sqrt{b}.$$

5. Arabic mathematicians knew various formulas for approximating the square root of natural numbers. For example:

$$\sqrt{T^2+r}\approx T+\frac{r}{2T+1},$$

where $T\in N,\ r\in N,\ r<T$.

Note that only in the 15th century did O. Cauchy generalize this formula for any natural numbers.

Conclusion

The use of historical and scientific materials in teaching mathematics has proven to be an effective method in deepening students' understanding of mathematical concepts. This approach not only makes the learning process more engaging but also provides context that connects theoretical knowledge with real-world applications. Studies have shown that such methods can significantly improve students' long-term retention and appreciation of mathematics[24]

Research suggests that incorporating historical methods into mathematics education not only improves students' conceptual understanding but also boosts their motivation and interest in the subject[25] [26].

Solving such problems helps increase students’ motivation and activity in studying mathematics and enrich their intellectual abilities.

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